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Probabilistic Study for Single Pile in Cohesionless Soil Using Monte Carlo Simulation Technique

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Abstract – The single pile capacity is mainly considered as a corner stone in the design of pile foundation system. The determination of more precise value of pile capacity will make a safe and economic design of pile foundation. The deterministic approaches are used for this purpose. The uncertainty associated with soil properties are taken into account by safety factor. In this paper, a probabilistic approach is used with the Monte Carlo simulation technique to consider the effect of uncertainty due to soil properties (unit weight and angle of internal friction). A single pile embedded in cohesionless soil has been analyzed. The results of the work reveal that the use of probabilistic approach will give more accurate results for prediction of pile capacity. Also, the statistical properties for the angle of internal friction are considered as the most effective on the statistical properties of the output results.

Index Terms— Monte Carlo simulation, Single pile capacity, Uncertainty of soil properties

1 Introduction

The design process of pile foundation system is affected mainly by the capacity of the single pile. The overestimate for the capacity of the single pile will cause a shear failure of the supporting soil(s) leading to serious damages of the supported structure. On the other hand, underestimate for the capacity of the single pile will lead to uneconomical design of the pile foundation. Hence, the accurate estimate for the capacity of the single pile is very important for the safe and economic design of the pile foundation. There are several method or procedures suggested to predict the capacity of the single pile based on the soil type (cohesive or cohesionless) and the method of installation of the pile (bored or driven) in addition to the pile geometry (Murthy, 2007).

Uncertainty caused by the randomness of natural phenomena or the implicit and inaccurate assumptions of the considered modeling approach are associated with many engineering problems (Morales, 2007).

There are three main sources of uncertainty in the foundation problems. These sources are the natural variability of soil mass, model uncertainty and parameters uncertainty. The soil properties depend on its location which leads to the first source of uncertainty. The second source of uncertainty is due to the lack of the proposed mathematical models to simulate the real behavior of the soil. Finally, the limiting number of soil samples and testing data will cause the parameters uncertainty (Shahin and Cheung, 2011).

All the procedures mentioned above are deterministic and they don't take into consideration the uncertainty in estimating pile capacity. They deal with the uncertainty by taking an appropriate amount of safety factor (Coduto, 2012).

Methods used to simulate the uncertainty are classified into three categories: analytical methods, approximate methods and Monte Carlo simulation. Analytical methods are considered computationally more effective, but they require some mathematical assumptions in order to simplify the problem. The approximate methods provide an approximate description of the statistical properties of output random variables such as first-order second-moment method (FOSMM) and point estimate methods. Monte Carlo simulation randomly generates values for uncertain input variables, and these values are taken into account to solve a deterministic problem (Morales, 2007).

In the present work, a probabilistic procedure has been used to study the effect of uncertainty due to soil properties on the estimated capacity for single pile driven in cohesionless soil. The procedure uses the Monte Carlo simulation technique to account the uncertainty of the soil properties. Parametric study is also conducted to investigate the effect of number of the iterations and coefficients of variation of soil properties used in simulation process on the simulation results.

2 Pile Capacity in Cohesionless Soil

In order to carry out the probabilistic simulation for pile capacity, a deterministic approach should be selected firstly. There are three main approaches are available to predict the pile capacity. Static approach uses the soil shear strength parameters to predict the pile capacity. While dynamic approach depends on the data of driving process to predict the pile capacity. The third approach uses the data of field tests (SPT and CPT) to estimate the pile capacity (Bowles, 1996).

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In this study, the static approach has been selected. The ultimate pile capacity comes from two components, the skin (friction) resistance and point (end bearing) resistance. The β-method is used among many methods to model the friction resistance of single pile. On the other hand, the traditional equation of ultimate bearing capacity of shallow foundation is used with some simplification to model the end bearing capacity of the pile. So, the capacity of a single pile subjected to axial load is:

$$Q_u = Q_s + Q_b \quad (1)$$

$$Q_s = \beta.\overline{\sigma}'_v.A_s \quad (2)$$

The val ues of (β) for driven pile is computed using the following expression suggested by Burland 1973 (Budha, 2007):

$$\beta = (1 - \sin\phi).\tan\phi \quad (3)$$

The ultimate pile end bearing capacity in any soil can be estimated by the ultimate bearing capacity of shallow foundation. The (N_γ) term is often neglected when pile base width is not large. So, the ultimate end bearing capacity of the pile embedded in a cohesionless soil is computed as:

$$Q_b = \sigma'_v.N'_q.A_b \quad (4)$$

The value of (N'_q) contains bearing capacity factor (N_q) and depth factor (d_q) as proposed by Hansen and Vesic'. It's determined from the following expression (Bowles, 1996):

$$N'_q = e^{\pi.\tan\phi}.\tan^2\left(45+\tfrac{\phi}{2}\right)\left\{1+2\tan\phi(1-\sin\phi)^2.\tan^{-1}\left(\tfrac{L}{B}\right)\right\} \quad (5)$$

3 Monte Carlo Simulation Technique

Monte Carlo simulation technique is considered as a powerful numerical technique useful for solving several complex problems (Murthy, 2000). As mentioned previously Monte Carlo simulation randomly generates values for uncertain input variables, and these values are taken into account to solve a deterministic problem. The main disadvantage of the Monte Carlo method is the great number of simulations required to achieve convergence. However, it uses deterministic routines to solve the problem in each iteration. Hence, Monte Carlo method is a special technique used to generate some numerical results without performing any physical testing (Nowak and Collins, 2000).

This technique has developed as a very powerful tool for engineers with only a knowledge of probability and statistics principles for evaluating the risk or reliability of complicated engineering systems (Haldar and Mahadevan, 2000).

Assume a variable (y) which is a function of some input variables (x1, x2, x3,….,xn). All input variables have uncertainty. The problem of determination of the statistical parameters and probability distribution of the output variable (y) can be solved using Monte Carlo simulation with the following six essential elements:

1. Define the problem in terms of all uncertain variables.
2. Quantify the probabilistic characteristics of all uncertain variables in terms of their PDFs (probability distribution functions).
3. Generate the values of these uncertain variables.
4. Evaluate the problem deterministically for each set of iteration of all the uncertain variables.
5. Extract probabilistic information from N such iterations.
6. Determine the accuracy and efficiency of the simulation.

The generated sample points (values of computed output) can be used to determine all the required sample statistics, the histogram, the frequency diagram, the PDF and the corresponding CDF.

4 Probabilistic Pile Capacity

In the present work, Monte Carlo simulation technique is used to simulate the uncertainty associated with the prediction of single pile capacity embedded in cohesionless soil. A precast concrete driven square pile is used. The prediction of pile capacity depends on the pile geometry (pile length and cross section of the pile) and soil properties (unit weight and angle of internal friction of the soil). Hence, there are four input variables. The pile length and the pile width are assumed to be deterministic for practical purposes. So, the uncertainty is associated with two input variables only. The simulation of angle of internal friction and the unit weight of the soil by their statistical parameters (mean and coefficient of variation). The statistical parameters of the angle of internal friction used in this study were suggested by Phoon and Kulhawy in 1999 (Phoon, 2004). While the statistical parameters of the soil unit weight were stated by Harr 1984 and Kulhway 1992 (Duncan, 2000). Table (1) shows all input variables and their parameters used in the present work.

Table 1 – PARAMETERS OF INPUT VARIABLES USED IN THE STUDY

Variable	Type	Statistical parameters		Distribution	Value
		Mean (μ)	Coefficient of variation (COV)		
Unit weight	Probabilistic	17.5	3% – 7%	Normal	-
Angle of internal friction	Probabilistic	30°	5% - 15%	Normal	-
Pile length	Deterministic	-	-	-	15 m
Pile width	Deterministic	-	-	-	28.5 cm

The steps of carrying out the simulation process to predict the pile capacity of a single pile subjected to axial load with uncertainty of two input variables are illustrated in the flow chart shown in Figure (1).

The simulation process is conducted using the package SimulAr v2.6 (Monte Carlo simulation in Excel). The steps to perform simulation with SimulAr are discussed in the following sections:

4.1 Define Input Variables

In this step, the input variables that will have random behavior in the future should be defined. Each of these variables will model by assigning a probability distribution reflecting their future behavior. Using of the historical information is one of the methods used to predict the behavior of the variables in the future. SimulAr includes 20 different types of probability distributions such as: normal, lognormal, triangular, uniform,…,etc.

In this study the soil unit weight and angle of internal friction are assumed the only variables that have uncertainty. The normal probability distribution is used to model their behavior. The normal distribution is generated with parameters mean (μ) and standard deviation (σ). These parameters are presented in Table (1). It should be noted that Table (1) contains the value of (COV) which is used to predict standard deviation based on the following:

$$COV = \frac{\sigma}{\mu} * 100 \quad (6)$$

The average value of the range of COV stated in Table (1) will select. The effect of variation of COV and in turn standard deviation (σ) will be investigated in other section.

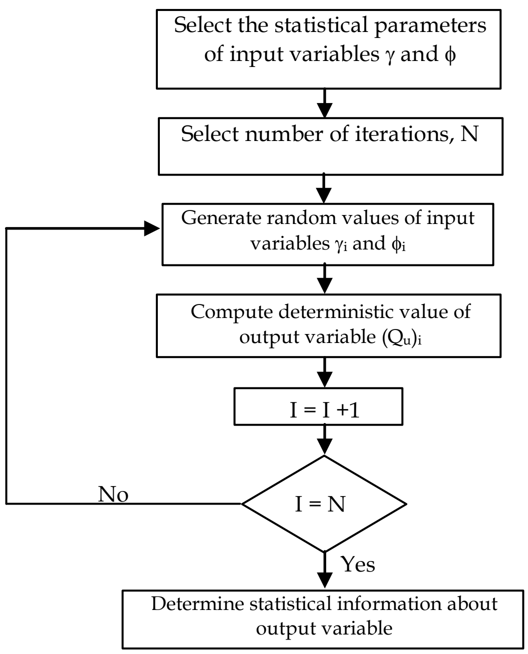

Figure (1) Flow chart used to conduct Monte Carlo simulation

4.2 Define Output Variables

After defining all input variables that have uncertainty in their behavior the simulation output variable(s) should be defined. The simulation output is a variable of unpredicted behavior.

The pile capacity (Qu) is selected to be the output variable of the simulation.

4.3 Run The Simulation

After inserting the input and output variables of the model, SimulAr will be ready to run simulation. In this stage the number of iterations is defined. The maximum number of iterations in SimulAr is (1000000).

Firstly, the number of iterations should be investigated to specify the number used through out this study. Four different models were run. These models with 100, 1000, 10000 and 100000 iterations. The statistical parameters of results of these models are presented in Table (2). Figure (2) illustrates the histograms for these models.

It can be noted from Figure (2) that the shape of histograms is not differ for different number of iterations. The cumulative probability of these models are demonstrated in Figure (3). It is clearly noticed based on Table (2) and Figure (3) that most statistical parameters are still without significant change after using 1000 iterations in simulation process and the cumulative probability curves are approximately the same. So, the number of iterations of (1000) will use thought out this study.

Table 2 – SOME STATISTICAL PARAMETERS OF SIMULATION OUTPUT FOR DIFFERENT NUMBER OF ITERATIONS

Statistical Parameter	Number of iterations, N			
	100	1000	10000	100000
Mean (μ)	1227.2	1236.0	1247.7	1246.0
Standard deviation (σ)	215.3	233.6	235.9	236.0
Kurtosis	0.526	1.762	1.983	1.854
Skewness	0.813	0.898	0.981	0.965
Coefficient of variation (COV %)	17.6	18.9	18.9	18.9

Also, it can be seen that all distribution histograms of the output data have a tail to the right (all skewness values are positive). The kurtosis values are greater than zero which are meant that the curves are peaked relative to the normal distribution curve.

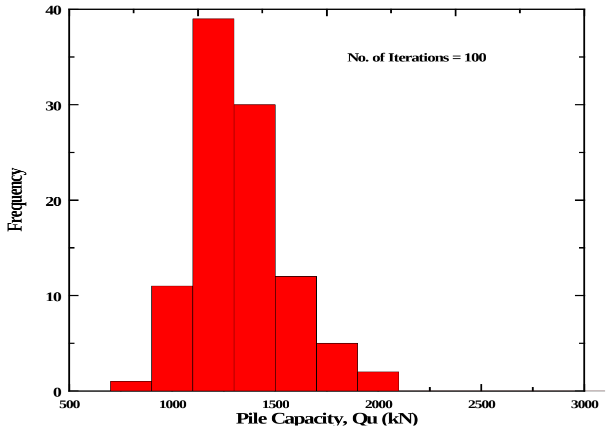

Figure (2) Histograms of output values of simulation model for different number of iterations

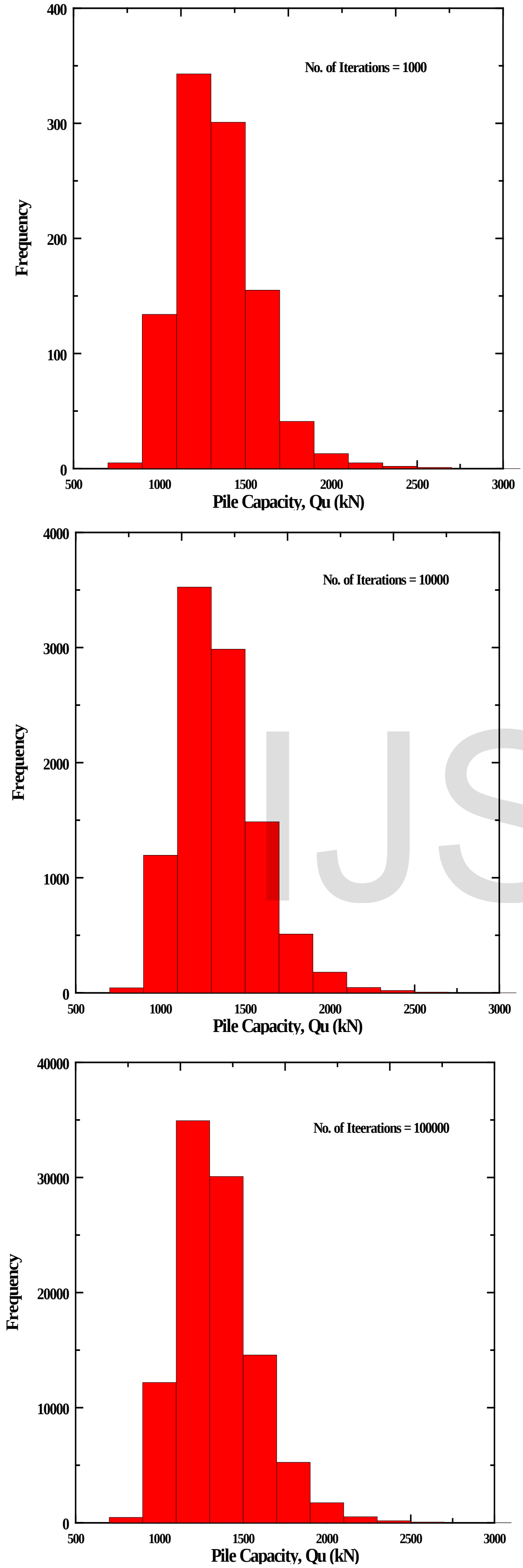

Figure (2) Continued

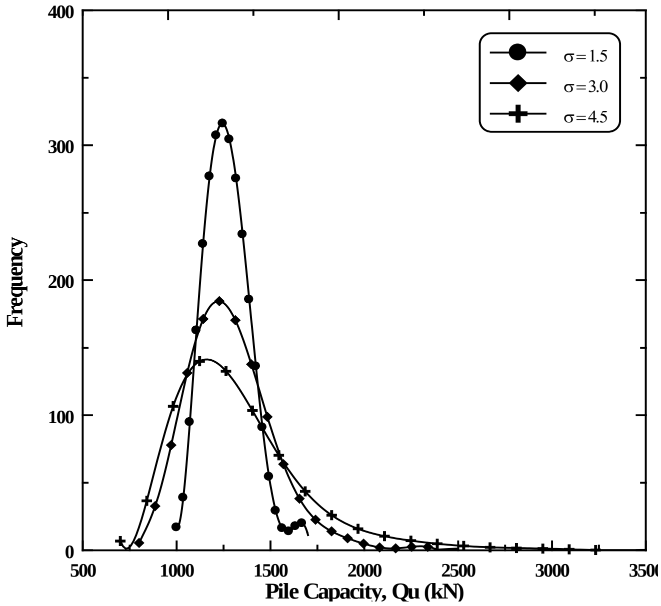

Figure (3) Variation of cumulative probability with single pile capacity for different number of iterations

5 Effect of Standard Deviation

In this section the effect of variations of standard deviations of the soil unit weight and angle of internal friction are studied. Three values of standard deviation are taken for each one (angle of internal friction and unit weight). The results of the simulation output values are as illustrated in Figures (4) and (5) respectively. It can be seen that the change for standard deviation of angle of internal friction will cause a significant change in the standard deviation of the output. The dissipation of the frequency distribution increases with the increasing of standard deviation. While, the increasing in the standard deviation of the soil unit weight doesn't affect the shape of the frequency distribution of the output results.

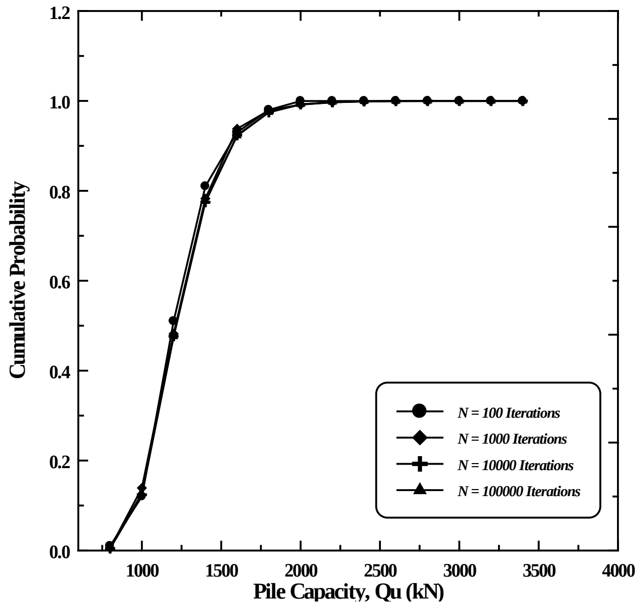

Figure (4) Frequency distribution single pile capacity for different values of standard deviation of soil angle of internal friction

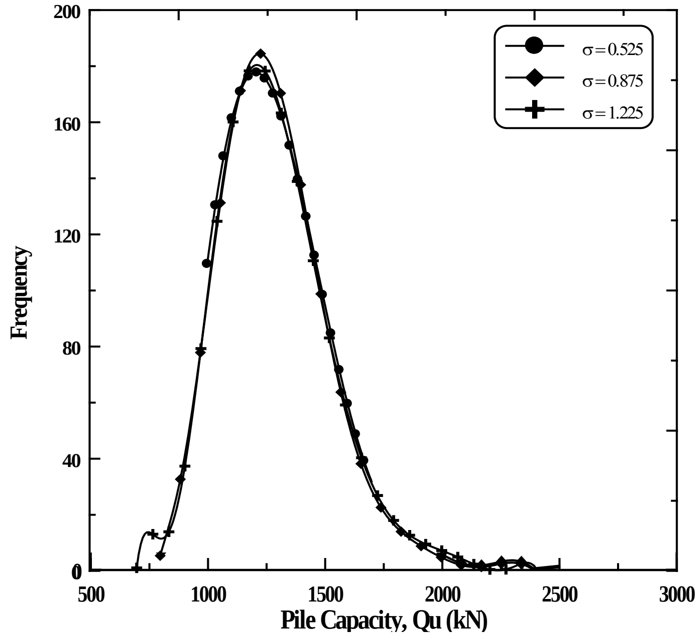

Figure (5) Frequency distribution single pile capacity for different values of standard deviation of soil unit weight

However, the variation of statistical properties of soil unit weight will not affect the statistical properties of the simulation results. This conclusion agree with the assumption stated by Shahin and Cheung 2011.

In the present work, the type of the distribution of the pile capacity resulting from simulation model is determined. Results of two distribution are investigated which are normal and lognormal distributions. Figure (6) shows the histogram of the output data with normal and lognormal presentation of them.

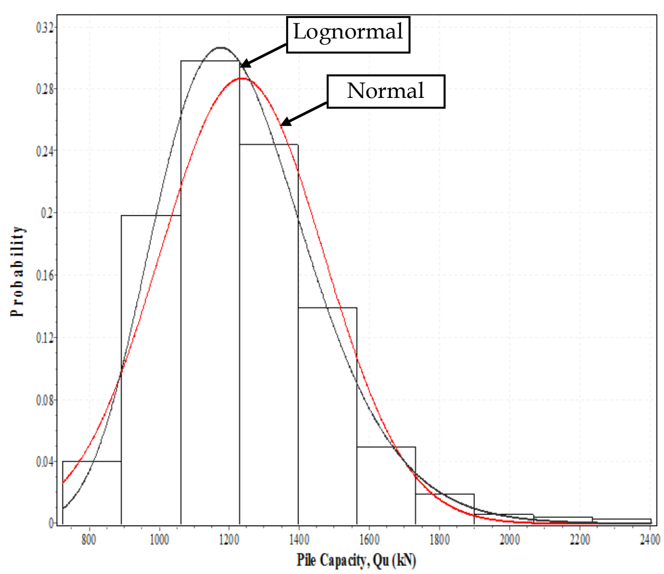

Figure (6) Histogram, normal and lognormal probability distributions of the single pile capacity

Three tests are used to measure the compatibility of these distributions with the simulation results. These tests are: Kolmogorov-Smirnov, Anderson-Darling and Chi-squared tests. All these tests have a null hypothesis (H_o) which is stated as the data follow the specified distribution. While, the alternative hypothesis (H_A) is that the data do not follow the specified distribution.

Table (3) demonstrates the results of the mentioned tests. It can be noted that the lognormal distribution is the most suitable distribution to simulate the results of the model for 5% significance level.

TABLE 3 - RESULTS OF GOODNESS OF STATISTICAL DISTRIBUTION FOR THE OUTPUT DATA

Type of distribution		Test		
		Kolmogorov-Smirnov	Anderson-Darling	Chi-squared
Normal	Statistic	0.0594	6.0404	30.415
	Critical value	0.0429	2.5018	16.919
	Accept H_o hypothesis?	No	No	No
Lognormal	Statistic	0.0219	0.4474	4.1853
	Critical value	0.0429	2.5018	16.919
	Accept H_o hypothesis?	Yes	Yes	Yes

6 Comparison of Deterministic and Probabilistic Results

A cumulative probability distribution with the pile capacity of the output results is illustrated in Figure (7).

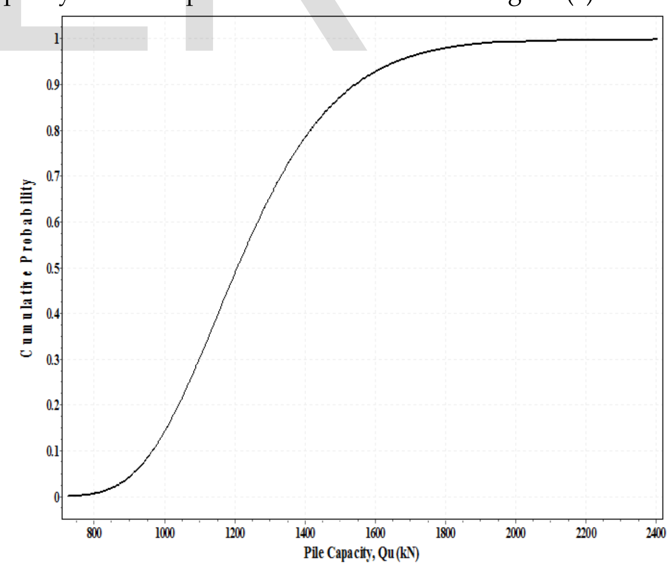

Figure (7) Cumulative probability distribution of the single pile capacity

A sample problem of precast concrete pile with 28.5 cm – width and total embedded length of 15 m is used. The soil have a unit weight (γ) of 17.5 kN/m^3 and angle of internal friction (ϕ) of 30^o. Using the equations (1) to (5) will give a deterministic ultimate pile capacity of 1214.4 kN. While, the probabilistic ultimate pile capacity for 90% and 95% confidence levels with the aid of the Figure (7) are 1542.7 kN and 1659.6 kN, respectively. Using Figure (7), the confidence level corresponding to the computed deterministic pile capacity is approximately 50%. It is clearly noticed that the use of deterministic approach gives more conservative result than that of probabilistic approach. Hence, the use of probabilistic approach in the prediction of pile capacity yields economical design of pile foundation system compared with the deterministic approach.

7 Conclusions

The probabilistic approach is used in the present study using Monte Carlo simulation technique to predict the capacity of single pile. Soil unit weight and angle of internal friction are considered the only factors that have uncertainty. Also, the effect of some parameters such as number of iterations and standard deviation of the soil properties has been studied. Based on the results of the work, it can be stated that:

1. The number of iterations used in the simulation doesn't have a significant effect on the output results.

2. The variation of standard deviation of the angle of internal friction affects significantly the output results. This effect will cause a high level of probability of failure. While, the variation of standard deviation of the soil unit weight will not affect the output results.

3. The probability distribution of the output of the model (pile capacity) is approximately lognormal distribution.

4. Using of deterministic approach with the aids of safety factor will give underestimate amount of pile capacity. The use of probabilistic approach is more appropriate to conduct safe and economic design.

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9 List of Symbols

A_b	Base area of pile
A_s	Surface area of pile
B	Width of pile section
COV	Coefficient of variation
L	Length of pile
N'_q	Bearing capacity of pile bearing capacity
Q_b	End bearing resistance of pile
Q_s	Friction resistance of pile
Q_u	Ultimate pile capacity
ϕ	Angle of internal friction of soil
μ	Mean
σ	Standard deviation
σ'_v	Effective overburden pressure at the pile tip
$\bar{\sigma}'_v$	Average effective overburden pressure along pile length